Tentamen EM2 22-11-1995

1a) $Z = R \,//\, R + \frac{1}{j\omega C} = R \,//\, \frac{1+j\omega RC}{j\omega C} = \dfrac{\frac{R(1+j\omega RC)}{j\omega C}}{R + \frac{(1+j\omega RC)}{j\omega C}} = \frac{R(1+j\omega RC)}{1+2j\omega RC}$

$V = I_0 Z = \frac{I_0 R(1+j\omega RC)}{1+2j\omega RC}$

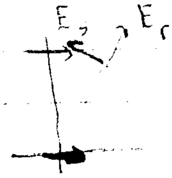
b) $V = V_0 \cos(\omega t + \varphi)$ met $V_0 = \frac{I_0 R\sqrt{1+\omega^2R^2C^2}}{\sqrt{1+4\omega^2R^2C^2}}$

en $\varphi = \arctan(\omega RC) - \arctan(2\omega RC)$

c) $I_C = \frac{V}{R+\frac{1}{j\omega C}} = \frac{Vj\omega C}{1+j\omega RC} = \frac{j\omega C(1+j\omega RC)\, I_0 R}{(1+2j\omega RC)(1+j\omega RC)} = \frac{j\omega C\, I_0 R}{1+2j\omega RC}$

2a) Veld ter plaatse van de dipool op $(0,d)$ t.g.v. het veld van de dipool op $(0,0)$

$E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$ met $\theta = \frac{\pi}{2}$ en $r = d$ $\rightarrow$ $E_r' = 0$



$E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$ met $\theta = \frac{\pi}{2}$ en $r = d$ $\rightarrow$ $E_\theta = \frac{p}{4\pi\varepsilon_0 d^3}$

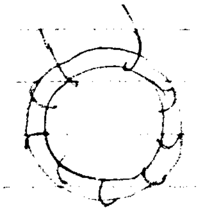
$\therefore E_x = -\frac{p}{4\pi\varepsilon_0 d^3}$ (t.g.v. de andere dipool)

$p = \alpha E = \alpha(E_0 + E_p)$

$\therefore p = \alpha\left(E_0 - \frac{p}{4\pi\varepsilon_0 d^3}\right) \rightarrow p + \frac{\alpha p}{4\pi\varepsilon_0 d^3} = \alpha E_0 \rightarrow p = \frac{\alpha E_0}{1 + \frac{\alpha}{4\pi\varepsilon_0 d^3}}$

3a) $\mu_r = \dfrac{L_m}{L_0}$ met L_0 zelfinductie van een spoel in vacuo en L_m zelfinductie van dezelfde spoel als de hele ruimte is gevuld met het permeabilium.

b)



$$\oint H.dL = NI$$

$$H\,2\pi R = NI \;\rightarrow\; H = \frac{NI}{2\pi R}$$

$$B = \mu_0\mu_r H \;\rightarrow\; B = \frac{\mu_0\mu_r NI}{2\pi R}$$

$$\mu_0 = 1{,}26\times10^{-6}\ \text{H/m}$$

$$\Bigg\}\; B = 1{,}2\ \text{T}$$

c) $$L = \frac{\Phi}{I} = \frac{NBA}{I} = \frac{\mu_0\mu_r N^2 A}{2\pi R} = 2{,}41\ \text{HT}$$

d) $$B = \mu_0(H+M) \rightarrow \mu_0\mu_r H = \mu_0(H+M) \rightarrow M = (\mu_r-1)H = \frac{(\mu_r-1)NI}{2\pi R}$$

$$\therefore M = 954\,293 = 9{,}5\times10^5\ \text{A/m}$$

4a) $$\int \vec{D}.d\vec{S} = Q \qquad \oint \vec{E}.d\vec{L} = -\frac{\partial}{\partial t}\oint \vec{B}.d\vec{S} \qquad \oint \vec{B}.d\vec{S} = 0$$

$$\oint \vec{H}.d\vec{L} = \int \vec{J}.d\vec{S} + \frac{\partial}{\partial t}\int \vec{D}.d\vec{S}$$

$$\vec{\nabla}.\vec{D} = \rho \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla}.\vec{B} = 0 \qquad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

b) $$P = \int_S N\,d\vec{S} = \int (\vec{E}\times\vec{H})\,d\vec{S}$$

$$= \int (\vec{E}\times\vec{H})\; 2\pi \underbrace{r\sin\theta}_{\text{straal ring}}\; \underbrace{r\,d\theta}_{\text{breedte ring}}$$

$$P = \int_0^{\pi/2} E_0H_0 \frac{\sin^2\theta\cos^2(\omega t - kr)}{r^2} \,.\, 2\pi r^2 \sin\theta\, d\theta \,.$$

$$P = 2\pi E_0H_0\cos^2(\omega t - kr)\int_0^{\pi/2}\sin^3\theta\, d\theta$$

$$\int_0^{\pi/2}\sin^3\theta\, d\theta = -\int_0^{\pi/2}(1-\cos^2\theta)\, d\cos\theta = -\left\{\left[\cos\theta - \tfrac{1}{3}\cos^3\theta\right]_0^{\pi/2}\right\} = \frac{2}{3}$$

$$\therefore \langle P\rangle = 2\pi E_0H_0\langle\cos^2(\omega t - kr)\rangle\,\frac{2}{3} \quad \therefore \langle P\rangle = \frac{2}{3}\pi E_0H_0$$